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MODELS OF TIME SERIES WITH SEASONAL FLUCTUATIONS IN THE FORECASTING OF PASSENGER TRAFFIC IN AIR TRANSPORT BASED ON THE STUDY OF WROCLAW AIRPORT

Abstract

Forecasting is one of the measures used in the planning process. It enables effective management of infrastructure and available human resources, including at airports. However, improper prediction of future trends may have economic consequences for the company. Therefore, the aim of the study is to determine forecasts of the number of passengers and air operations characterized by the smallest deviations from the real values, on the example of Wrocław Airport. For the needs of the study, an analysis of econometric models of seasonal fluctuations was carried out, as well as the method of seasonality indicators and homologous period trends. The methods were selected due to the low level of forecast error using expired forecasts for 2017. The study was completed with the choice of a method generating forecasts burdened with the smallest error.

Keywords: econometric models of seasonal fluctuations, seasonal indices, homologous period trend, air transport

Introduction

The efficiency of infrastructure management and disposable human resources in business units is possible, among others thanks to proper forecasting, being one of the elements used in the planning process. Therefore, the aim of the study was to determine forecasts of the number of passengers and air operations characterized by the smallest deviations from the real values, on the example of Wrocław Airport.

Due to the occurrence of seasonal fluctuations, forecasting of passenger traffic in air transport is more complicated than in the case when these fluctuations do

not occur. In addition, it is worth noting that “isolating the impact of seasonal fluctuations on the shaping of the forecasted phenomenon and its inclusion in the forecasting process raises the precision of predictions”¹.

The study analyzed econometric models of seasonal variations, using the seasonality method and the trend of homologous periods. Quarterly data from the years 2012–2017 made available by the Civil Aviation Office and the analyzed airport were used. Based on the indicated methods, forecasts of expired passenger numbers and the number of flight operations for 2017 were made. While verifying the ex post forecast, it was assumed that the obtained level of forecast errors is the justification for expanding it to the next 2018.

1. Methodology

In econometric models of seasonal fluctuations, some dummy variables are introduced, which correspond to the distinguished phases of the cycle. The estimated coefficients located at these variables are therefore measures of cyclical effects². The general record of the model with the linear trend and the periodic seasonal component is as follows³:

$$\hat{y} = \alpha_0 + \alpha_1 t + \sum_{k=1}^m d_{0k} Q_{kt} + U_t \text{ maintaining the condition: } \sum_{k=1}^m d_{0k} = 0,$$

where:

α_0, α_1 – model parameters,

d_{0k} – model parameters reflecting fixed parts of seasonal effects in individual phases of the cycle,

Q_{kt} – a dummy variable taking values equal to 1 in periods/moments corresponding to k-th phase of the cycle and equal to 0 in periods/moments corresponding to other phases of the cycle,

k – variable specifying the number of the seasonal cycle,

U_t – a random component.

In addition to the above, the study uses the method of seasonality indicators⁴ and the method of trends of homologous periods⁵. Based on the indicated methods, the process of determining expired forecasts was carried out (the expired

¹ P. Dittmann, *Prognozowanie w przedsiębiorstwie. Metody i ich zastosowanie*, Oficyna Wydawnicza, Cracow 2003, p. 83.

² Z. Pawłowski, *Ekonometria*, PWN, Warsaw 1966, p. 161.

³ *Zastosowanie hierarchicznych modeli szeregów czasowych w prognozowaniu zmiennych ekonomicznych z wahaniami sezonowymi*, ed. J. Zawadzki, Agricultural University of Szczecin Publisher, Szczecin 2003, p. 9–10; P. Dittmann, *Prognozowanie w przedsiębiorstwie...*, p. 129.

⁴ Full methodology in: A. Barczak, *Pomiar wahań sezonowych ruchu pasażerskiego na przykładzie Portu Lotniczego Gdańsk*, Folia Pomeranae Universitatis Technologiae Stetinensis. Oeconomica 2015, 321(80)3, p. 5–14.

⁵ Full methodology in: A. Barczak, *Metoda trendów jednoimiennych okresów jako narzędzie prognozowania ruchu pasażerskiego na przykładzie Portu Lotniczego Gdańsk* [in:] *Wybrane zagadnienia logistyki stosowanej*, vol. 4, ed. J. Feliks, AGH University of Science and Technology Publisher, Cracow 2016, p. 13–24.

forecast is a forecast that is “determined for the t time, for which the real value of the forecast variable is known”⁶) and subsequently – forecasts for 2018.

2. Forecasting

For all estimated econometric models of seasonal fluctuations and in the case of trends of homologous periods, the type of model was selected based on graphical analysis and the analysis of increments for numerical data on passenger traffic at Wrocław Airport. The basis for adopting the models for further analysis was to meet the following conditions:

- the determination coefficient in the range (0.5625;1);
- the random variation coefficient not exceeding 15%;
- randomness of the rest of the estimated models (tested using a series test).

First of all, the results obtained using econometric models of seasonal fluctuations were depicted. The model of the time series, taking into account seasonality for the number of passengers, has the form:

$$\hat{y} = 437746.1531 + 11710.2688t - 121837.2844Q_1 + 33084.9469Q_2 + 153489.9281Q_3 - 64737.5906Q_4$$

where:

$$R^2 = 0.99017,$$

$$Vs = 3.40\%^8.$$

Table 1 presents estimated forecasts for 2017 for the number of passengers using the data for 2013–2016 together with forecasting errors⁹.

Table 1. Forecasts expired for 2017 using the time series analysis with regard to seasonality for the number of passengers

Period	Forecast	Absolute error forecast ex post	Relative forecast error ex post (%)
I quarter	514 983	42 312	7.59
II quarter	681 616	59 052	7.97
III quarter	813 731	100 668	11.01
IV quarter	607 214	35 495	5.52

Source: own elaboration based on: by Civil Aviation Authority in Poland (<http://www.ulc.gov.pl/pl/regulacja-rynku/statystyki-i-analazy-rynku-transportu-lotniczego> – access: 20.02.2018) and Airport Wrocław (<http://airport.wroclaw.pl/lotnisko/o-lotnisku/statystyki/> – access: 20.02.2018)

⁶ M. Cieślak, *Organizacja procesu prognostycznego* [in:] *Prognozowanie gospodarcze. Metody i zastosowania*, ed. *idem*, PWN, Warsaw 1997, p. 56.

⁷ Coefficient of determination.

⁸ Random variation coefficient.

⁹ Full methodology in: A. Barczak, A. Nurzyńska, S. Król, *Transport we współczesnej gospodarce – wybrane aspekty*, Sophia Scientific Publisher, Katowice 2017, p. 49–50.

For the number of air operations, the estimated model of the time series, taking into account seasonality, is:

$$\hat{y} = 4490.9188 + 101.8625t - 621.7063Q_1 + 279.4313Q_2 + 864.3188Q_3 - 522.0438Q_4,$$

where:

$$R^2 = 0.8007,$$

$$Vs = 11.00\%.$$

Table 2 presents the forecasts of the number of aviation operations designated for 2017. Data for 2013–2016 were used and ex-post prediction errors were determined.

Table 2. Forecasts expired for 2017 using the time series analysis with regard to seasonality for the number of aviation operations

Period	Forecast	Absolute error forecast ex post	Relative forecast error ex post (%)
I quarter	5601	361	6.06
II quarter	6604	489	6.89
III quarter	7291	819	10.10
IV quarter	6006	565	8.60

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

After assuming that a forecast error of up to 10% of the real value is admitted, the analysis of the absolute error of the ex post forecast and the relative error of the ex post forecast shows that the expired forecasts are characterized by relatively small deviations from the actually observed number of passengers and the number of operations at Wrocław Airport, excluding the third quarter. Therefore, time series models were estimated with regard to seasonality, successively for the number of passengers and flight operations using data for the years 2013–2017:

$$\hat{y} = 418447.125 + 14679.35t - 120797.675Q_1 + 34503.575Q_2 + 160262.625Q_3 - 73968.525Q_4,$$

where:

$$R^2 = 0.9753,$$

$$Vs = 4.52\%,$$

$$\hat{y} = 4309.3656 + 129.7938t - 619.3094Q_1 + 279.4969Q_2 + 902.5031Q_3 - 562.6906Q_4,$$

where:

$$R^2 = 0.8664,$$

$$Vs = 7.93\%.$$

On this basis, quarterly forecasts of both variables for 2018 were calculated, excluding the third quarter (Table 3).

Table 3. Forecasts for particular quarters of 2018, including errors¹⁰, using the time series analysis, taking into account seasonality for the number of passengers and flight operations

Period	Forecast	Average prediction error S_T^p	Relative forecast error ex ante η_T (%)
Numer of passesngers			
I quarter	605 916	28 539.5792	4.71
II quarter	775 896	28 925.2667	3.73
IV quarter	696 783	29 783.2600	4.27
Number of air operations			
I quarter	6 416	495.8887	7.73
II quarter	7 444	502.5902	6.75
IV quarter	6 862	517.4983	7.54

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

The analysis of the error of the average prediction and the relative error of the ex ante forecast shows that the estimated forecasts are characterized by small deviations from the actually observed values. The lowest relative prediction error of 3.73% was obtained in the second quarter for the number of passengers, and the highest was 7.73% in the first quarter for the number of air operations.

The second method used is the seasonal factors method. In order to conduct the forecasting process for each variable, empirical data charts were determined. On this basis, it was possible to conclude on the course of seasonal fluctuations. In each case, they were series with occurring periodic fluctuations with an amplitude having a growing character over time which indicates multiplicative properties. Therefore, the function of the exponential trend was determined for quarterly data on the number of passengers served and flight operations. Subsequently, corrective coefficients were determined to change the raw seasonality indicators into cleared ones. Then the values of forecasts expired for 2017 were estimated and ex post forecast errors were determined (Table 4).

Table 4. Forecasts expired for 2017 using the seasonality ratio method for the number of passengers and flight operations

Period	The form of the trend function	Forecast	Absolute error forecast ex post	Relative forecast error ex post (%)
Number of passengers				
I quarter	$\hat{y} = 416503.2206 e^{0.0268t}$	505 052	52 243	9.37
II quarter		720 867	19 801	2.67
III quarter		892 004	22 395	2.45
IV quarter		623 349	19 360	3.01

¹⁰ Full methodology in: A. Barczak, *Wykorzystanie wybranych metod ilościowych w analizie pasażerskiego ruchu lotniczego w Polsce*, Scientific Papers of University of Economics in Wrocław, Economics 2015, 401, p. 26–35.

Table 4. cont.

Period	The form of the trend function	Forecast	Absolute error forecast ex post	Relative forecast error ex post (%)
Number of air operations				
I quarter	$\hat{y} = 4498.6281 e^{0.0190t}$	5 480	-482	8.09
II quarter		6 667	-426	6.00
III quarter		7 496	-614	7.57
IV quarter		5 946	-625	9.51

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

The analysis of the determined seasonality ratios indicates that in the examined period (2013–2016) only due to seasonality the number of passengers in the first quarter was lower compared to the trend by 23.11% on average, in the second quarter higher by 6.84%, third quarter higher by 28.71%, and in the fourth quarter lower by 12.43%. In the case of the number of flight operations, due to seasonality, their number compared to the trend in the first quarter was lower on average by 11.81%, in the second quarter higher by 5.28%, in the third quarter – by 16.14%, while in the fourth quarter, it was lower by 9.61%.

Due to the fact that this method of forecasting is not based on a formal model and it is not possible to determine the forecast error¹¹ in Table 5, it was limited to presenting the estimated trend functions and forecasts for 2018.

Table 5. Forecasts for individual quarters of 2018 using the seasonality ratio method for the number of passengers and flight operations

Period	The form of the trend function	Forecast
Number of passengers		
I quarter	$\hat{y} = 412017.8542 e^{0.0283t}$	581 974
II quarter		818 848
III quarter		1 012 904
IV quarter		708 438
Number of air operations		
I quarter	$\hat{y} = 4389.8156 e^{0.0228t}$	6 292
II quarter		7 617
III quarter		8 596
IV quarter		6 846

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

The analysis of seasonality ratios, using the data for 2013–2017 indicates that in the analyzed period, the number of passengers in the first quarter was lower by 22.04% in the first quarter as compared to the trend, 6.63% higher in the second quarter, in the third quarter it was higher by 28.22%, while in the fourth quarter it was lower by 12.82%. As a result of seasonality, in the case of the number of operations, compared to the trend, their number in the first quarter was lower on average

¹¹ J. Józwiak, J. Podgórski, *Statystyka od podstaw*, PWE, Warsaw 2009, p. 441–442.

by 11.21%, in the second quarter by 5.08%, in the third quarter – by 15.90%, while in the fourth quarter, 9.77% lower.

Another method used for the needs of the study is the method of trends of homologous periods. The estimated trend models for individual quarters, along with the basic adjustment measures are presented in Tables 6–7.

Table 6. Trends of homologous periods for the number of passengers (2012–2016)

Period	The type of trend	The form of the function	R ²	V _s (%)
I quarter	exponential	$\hat{y} = 283395.8606 e^{0.0978t}$	0.8996	5.65
II quarter	quadratic	$\hat{y} = 644628 - 94740.8571t + 18890.1429t^2$	0.7772	5.00
III quarter	linear	$\hat{y} = 587678.2 + 32299.2t$	0.9184	2.57
IV quarter	exponential	$\hat{y} = 348362.889 e^{0.0971t}$	0.9669	3.05

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Table 7. Trends of homologous periods for the number of flight operations (2012–2016)

Period	The type of trend	The form of the function	R ²	V _s (%)
I quarter	quadratic	$\hat{y} = 5211.2 - 783.8857t + 155.7143t^2$	0.9157	2.88
II quarter	quadratic	$\hat{y} = 8384.4 - 2339.9143t + 399.2857t^2$	0.9735	2.49
III quarter	quadratic	$\hat{y} = 7145.8 - 1150.7429t + 231.8571t^2$	0.8199	5.00
IV quarter	quadratic	$\hat{y} = 5377.6 - 904.3571t + 203.6429t^2$	0.8172	7.02

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Tables 8–9 contain expired forecasts, including errors, for the number of passengers and the number of flight operations, respectively.

Table 8. Forecasts expired for 2017 with the use of trends of the homologous periods for the number of passengers

Period	Forecast	Absolute error forecast ex post	Relative forecast error ex post (%)
I quarter	509 609	47 686	8.56
II quarter	756 228	-15 560	-2.10
III quarter	781 473	132 926	14.54
IV quarter	623 809	18 900	2.94

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Table 9. Forecasts expired for 2017, including errors, using trends of the homologous periods for the number of operations

Period	Forecast	Absolute error forecast ex post	Relative forecast error ex post (%)
I quarter	6114	-152	-2.55
II quarter	8719	-1626	-22.92
III quarter	8588	-478	-5.89
IV quarter	7283	-712	-10.84

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Again assuming that a forecast error of up to 10% of the real value is admitted, using the method of the trends of the homologous periods (data for 2013–2017) for the first, second and fourth quarter for the number of passengers and for the first and third quarter for operations was determined regarding the air operations (Table 10). For these periods, the forecast for 2018 was set, as well as the error of the average prediction and the relative error of the ex ante forecast (Table 11).

Table 10. Trends of the homologous periods for the number of passengers and air operations (2013–2017)

Period	The type of trend	The form of the function	R ²	V _s (%)
Number of passengers				
I quarter	exponential	$\hat{y} = 284775.8461 e^{0.1313t}$	0.9745	3.58
II quarter	exponential	$\hat{y} = 447111.5803 e^{0.0947t}$	0.9516	4.01
IV quarter	linear	$\hat{y} = 349308.3 + 57107.5t$	0.9852	2.45
Number of air operations				
I quarter	exponential	$\hat{y} = 3757.7368 e^{0.0828t}$	0.8181	6.90
III quarter	linear	$\hat{y} = 4899.6 + 580t$	0.7806	8.46

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Table 11. Forecasts for selected quarters of 2018, including errors, using trends of homologous periods for the number of passengers and flight operations

Period	Forecast	Average prediction error S_T^p	Relative forecast error ex ante η_T (%)
Number of passengers			
I quarter	626 095	22273.6138	3.56
II quarter	789 191	34893.7772	4.42
IV quarter	691 953	18516.09	2.68
Number of air operations			
I quarter	6 176	485.9070	7.87
III quarter	8 380	813.6230	9.71

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Conclusions

In the case of econometric models of seasonal fluctuations, the highest errors of ex post forecasts were obtained in the third quarter. The situation is similar for the forecast of the expired passenger population based on the trends of the homologous periods. In the case of the number of air operations, the highest error was in the second and fourth quarter. It should be emphasized that for expired forecasts generated using the seasonality ratio method, the relative error of the ex post forecast did not exceed the 10% threshold.

Based on the values of relative errors, ex post forecasts can indicate the values forecast for individual quarters of 2018 (Table 12).

Table 12. Forecasts for particular quarters of 2018 for the number of passengers and flight operations – an overview of methods

Period	Number of passengers – forecast	Number of air operations – forecast
I quarter	605 916*	6 176***
II quarter	789 191***	7 617**
III quarter	1 012 904**	8 380***
IV quarter	691 953***	6 862*

* econometric model of seasonal fluctuations, ** seasonal indices method, *** homologous period trend

Source: own elaboration based on: by Civil Aviation Authority in Poland and Airport Wrocław

Summarizing, the analysis of errors of expired forecasts indicates that the most reliable forecasts can be obtained using the trend method of homologous periods.

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