AN ATTEMPT TOWARDS A MODEL APPROACH TO CHOOSING A STOCK REPLENISHMENT SYSTEM UNDER CONDITIONS OF INDEPENDENT DEMAND

Abstract

The premise for approaching the subject is the need to support companies in selecting the best stock replenishment system, taking into account all external or internal limitations related to delivery terms and order volumes assuming that safety stock is the central factor ensuring the achievement of the intended availability level. In order to meet the objective, several stock replenishment systems are briefly described. Then, prerequisites for their proper selection are identified and classified. As a result, a selection table is developed along with an algorithm for selecting an optimal stock replenishment system based on an economic or qualitative criterion.

Keywords: inventory management, stock replenishment systems

Introduction

The approach presented in the paper was developed by the author in order to aid enterprises in choosing the right system of stock replenishment and determining the principles of estimating the parameters controlling those systems as part of the consultation work of the Institute of Logistics and Warehousing. It was also used in the doctoral dissertation of one of its faculty members (Kolińska, 2015).

The two basic questions that arise when designing stock replenishment systems are: when to place orders and what should be the size of the order. The answer is provided directly or indirectly by the parameters controlling stock replenishment systems. There exist numerous such systems and the implementation of the most of them is backed by IT systems supporting the management of the flow of goods available in the market (Pyrek, 2007).
A correct calculation of control parameters, which answer the above questions, is usually given crucial importance. These calculations have to take into account a number of conditions connected with demand and quantities that characterize the process of stock replenishment:

- random variability of demand – its scale and character (including the type of frequency distribution),
- random variability of replenishment lead time – its scale and character,
- quantitative and qualitative deficiencies in deliveries, damage made during acceptance of delivery, losses during storage – also in statistical terms, as well as the criteria related to the availability of stock and its replenishment costs:
  - service level – its value according to the assumed type of indicator – probabilistic or quantitative approach (Tempelmeier, 2000, pp. 361–380),
  - the costs of replenishing, maintaining, and lack of stock.

Very often, however, the essential issue is the very choice of a system, determined by organizational and technical constraints on the part of both the provider and the recipient, as well as the above-listed criteria.

1. Review of stock replenishment systems

The necessity to react to the above-mentioned random variability of demand requires employing either a “fixed quantity, variable cycle” system or a “fixed cycle, variable quantity” system. In both cases the aim is to adjust the rhythm and size of orders to the observed random variability of demand.

<table>
<thead>
<tr>
<th>Order quantity</th>
<th>FIXED</th>
<th>VARIABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BQ – system based on re-order level (re-order point)</td>
<td></td>
<td></td>
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<tr>
<td>sQ – periodic review with decision level s and fixed order quantity Q</td>
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<tr>
<td>QT system (fixed-quantity orders placed at fixed intervals)</td>
<td></td>
<td></td>
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<tr>
<td>ST – system based on periodic review</td>
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</tbody>
</table>

Figure 1. Classification of stock replenishment systems based on safety stock
Source: (own elaboration)
This approach can be generally presented in the form of a simple matrix which shows the most important stock replenishment systems corresponding to the discussed cases. The symbols of the systems are based on the terminology developed by the European Logistics Association (ELA, 1994).

The paper examines five selected systems presented in Figure 1. Their implementation rules are discussed below.

1) BQ system – based on re-order level (re-order point). Order is placed in a fixed quantity Q when, after another operation (including reservations), the quantity of economic stock \( S_E \) becomes equal to or less than the re-order level \( B \). Each time, economic stock is calculated as follows:

\[
S_E = S_W + S_O - S_R
\]  

(1)

where:

- \( S_W \) – warehoused stock,
- \( S_O \) – stock on order, but not yet delivered,
- \( S_R \) – stock requisitioned, but not yet issued.

![Figure 2. Stock replenishment in BQ system](own elaboration)

2) ST system – based on periodic review. Orders are placed at fixed intervals \( T_0 \), in a quantity equal to the difference between maximum stock level \( S \) and current quantity of economic stock \( S_E \).

![Figure 3. Stock replenishment in ST system](own elaboration)
3) BS system – MIN-MAX system based on re-order level. Order is placed when, after another operation, the quantity of economic stock $S_E$ becomes equal to or less than re-order level $B$, in a quantity equal to the difference between maximum stock level $S$ and current quantity of economic stock $S_E$.

![Figure 4. Stock replenishment in BS system](Image)

Source: (own elaboration)

4) sS system – MIN-MAX system based on periodic review. Stock review is conducted at fixed intervals ($T_o$), but orders are placed only if the quantity of economic stock $S_E$ at the time of review is equal to or less than re-order level $s$, in a quantity equal to the difference between maximum stock level $S$ and current quantity of economic stock $S_E$.

![Figure 5. Stock replenishment in sS system](Image)

Source: (own elaboration)

5) sQ system – system based on interim review. Stock review is conducted at fixed intervals ($T_o$), but orders in a fixed quantity $Q$ are placed only if the quantity of economic stock $S_E$ at the time of review is equal to or less than re-order level $s$. 

![Figure 5. Stock replenishment in sS system](Image)
2. Limitations in selecting a stock replenishment system

The process of selecting a suitable stock replenishment system must be conducted in two stages. First, all organizational or technical limitations on the part of both the provider and the customer that could exclude some of the systems under consideration should be identified. Those limitations can be related to the two crucial issues of when and how much to order. To determine that, the following three questions must be answered:

- Do orders have to be placed at fixed intervals (e.g. once a month, once a quarter) \( T_o \), or can they be placed at any given time?
- Can order quantity \( Q \) be variable?
- Is there a minimum order quantity (either imposed by the supplier or determined by internal arrangements) \( Q_{\text{min}} \)?

Assuming that the only possible answers to the above questions are “yes” and “no,” Table 1 summarizes all possible variants of answers, at the same time indicating those sequences of answers that rule out particular stock replenishment systems. It has also been assumed that, e.g., a positive answer to the question “Can order quantity \( Q \) be variable?” does not rule out solutions with fixed order quantities (it can, but does not have to be variable).

Based on the limitations to implementing the stock replenishment systems under consideration identified in Table 1, suitable systems for each variant of limitations (or lack thereof) are presented in Table 2.

Based on Table 2 as well as on the characteristics of all five stock replenishment systems under consideration Table 3 was constructed, which presents the connections between particular systems, their control parameters, and quantities resulting from qualitative (service level), economic (economic order quantity \( EOQ \), economic review cycle \( T_{oeq} \)), or imposed constraints (minimum order quantity \( Q_{\text{min}} \), imposed review cycle \( T_{oi} \)).

\[ n = 2^3 = 8. \]
Table 1. Possible variants of answers to the questions identifying the limitations to implementing the stock replenishment systems under consideration

<table>
<thead>
<tr>
<th>Is there a fixed order interval $T_O$ imposed?</th>
<th>Can order quantity $Q$ be variable?</th>
<th>Is there a minimum order quantity ($Q_{min}$) set?</th>
<th>BQ</th>
<th>ST</th>
<th>BS</th>
<th>sS</th>
<th>$sQ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO</td>
<td>NO</td>
<td>NO</td>
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</tbody>
</table>

Source: (own elaboration)
Table 2. Suitable stock replenishment systems for each variant of limitations

<table>
<thead>
<tr>
<th>Is there a fixed order interval $T_0$ imposed?</th>
<th>Can order quantity $Q$ be variable?</th>
<th>Is there a minimum order quantity $(Q_{min})$ set?</th>
<th>BQ</th>
<th>ST</th>
<th>BS</th>
<th>sS</th>
<th>sQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
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<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

Source: (own elaboration)

Table 3. Connections between particular systems, their control parameters, and quantities resulting from qualitative, economic, or imposed constraints

<table>
<thead>
<tr>
<th>Circumstances ↓</th>
<th>Parameters →</th>
<th>Systems</th>
<th>Q</th>
<th>B</th>
<th>$T_0$</th>
<th>S</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic order quantity EOQ considering that $EOQ\geq Q_{min}$</td>
<td>BQ</td>
<td>BS</td>
<td>sQ</td>
<td>sS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Service level SL</td>
<td>BQ</td>
<td>BS</td>
<td>sQ</td>
<td>sS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economic review cycle $T_{oec}$ considering the limitation related with an imposed review cycle $T_{oN}$</td>
<td>ST</td>
<td>sQ</td>
<td>sS</td>
<td>sS</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: (own elaboration)

Based on Tables 2 and 3, for each system usable under given constraints sets of control parameters and quantities that affect their optimal values were identified, taking into account the qualitative, economic, and imposed constraints.

Table 4. Sets of control parameters of the stock replenishment systems under consideration and quantities that affect their optimal value

<table>
<thead>
<tr>
<th>Is there a fixed order interval $T_0$ imposed?</th>
<th>Can order quantity $Q$ be variable?</th>
<th>Is there a minimum order quantity $(Q_{min})$ set?</th>
<th>SYSTEM</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>Q</td>
<td>B</td>
</tr>
<tr>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>BQ</td>
<td>EOQ [SL]</td>
</tr>
<tr>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>ST</td>
<td>$T_{ol}$ [SL]</td>
</tr>
<tr>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>sS</td>
<td>$T_{ol}$ [SL, $T_{ol}$]</td>
</tr>
</tbody>
</table>
3. General principles of selecting an optimal stock replenishment system

Having identified all limitations to selecting a suitable system as well as their control parameters and quantities affecting those parameters, an attempt can be made to select an optimal system. Figure 7 presents the subsequent steps that lead to this decision and the connections between them:

1) identifying input data that includes:
   - historic data about demand and replenishment lead time that allow to determine key parameters describing the variability of demand in replenishment lead time ($\sigma_{DLT}$): average demand $D$, its standard deviation $\sigma_D$, average replenishment lead time $LT$, and its standard deviation $\sigma_{LT}$;
   - limitations and requirements imposed on the process;
2) determining the set of acceptable solutions (systems) along with optimal (acceptable) values of control parameters;
3) determining the criterion for choosing the particular system from this set;
4) choosing the optimal system according to the adopted selection criterion.

The adopted selection criterion can be of various nature. One of the basic criteria is undoubtedly the cost criterion, where the objective function is the total replenishment, holding, and stock-out cost:

$$TSC = FRC + VRC + FHC + VHCoSS + VHCoCS + SoC_1 + SoC_2$$  \(2\)

where:
FRC – fixed replenishment cost (independent from the frequency and number of deliveries),
VRC – variable replenishment cost (dependent on the frequency and number of deliveries),
FHC – fixed holding cost (independent from its quantity),
VHCoSS – variable holding cost of safety stock (dependent on its quantity),
VHCoCS – variable holding cost of cycle stock (dependent on its quantity),
SoC₁ – stock-out cost depending on the probability of shortages in replenishment lead time,
SoC₂ – stock-out cost depending on the number of shortages in the given period.

The particular cost components, especially those associated with stock replenishment – both fixed and variable – can, generally speaking, depend not only on the values of control parameters, but also on the stock replenishment system itself. For instance, review costs in BQ and BS systems can be higher than in ST, sQ, and sS systems. These costs also depend on the mutual dependence of control parameters (Krzyżaniak, 2016, pp. 59–72).

Another criterion can be related to quality measured by service level (stock availability) – not so much the value of this parameter (as it will usually constitute one of the limitations), as, for instance, the sensitivity of the actual service level to various factors – e.g. in sS (Krzyżaniak, Fechner, 2013, pp. 127–142) and BS (Krzyżaniak, 2015).

The choice can also be based on a multi-criteria approach (Reszka, 2014).

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Figure 7. Graphic illustration of the steps leading to the selection of an optimal stock replenishment system
Source: (own elaboration)
Conclusions

The paper presents a model approach to choosing an optimal stock replenishment system. The said approach consists of several steps. The first step is to identify the limitations imposed on stock replenishment based on the answers to three questions regarding the quantity and frequency of orders. The answers narrow the set of applicable systems. The article discusses five basic systems: BQ, ST, BS, sQ, and sS.

Control parameters are determined for acceptable systems, taking into account qualitative (required service level) or economic criteria (e.g. based on economic order quantity).

Thus selected and parametrized systems should be then compared based on the chosen economic or qualitative criteria (or their chosen combination) and the best stock replenishment system under given circumstances should be recommended based on the results of this comparison.

The presented approach has been – to some extent – used in practice for selecting optimal spare parts replenishment system of a manufacturing company.

References

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